Correction TD2 (exercice 1)

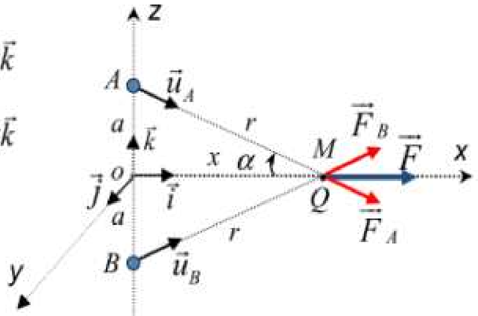
Exercice 1 :

1) Force électrostatique $\vec{F}$ qu'exercent q_A et q_B sur la charge Q

$q_A = q_B = q$; $OA = OB = a$ et $r_A = r_B = r$

q_A exerce sur Q $\vec{F_A} = \frac{1}{4\pi\varepsilon_0}\frac{q.Q}{r_A^2}\vec{u}_A$; $\vec{u}_A = \cos\alpha\vec{i} - \sin\alpha\vec{k}$

q_B exerce sur Q $\vec{F_B} = \frac{1}{4\pi\varepsilon_0}\frac{q.Q}{r_B^2}\vec{u}_B$; $\vec{u}_B = \cos\alpha\vec{i} + \sin\alpha\vec{k}$

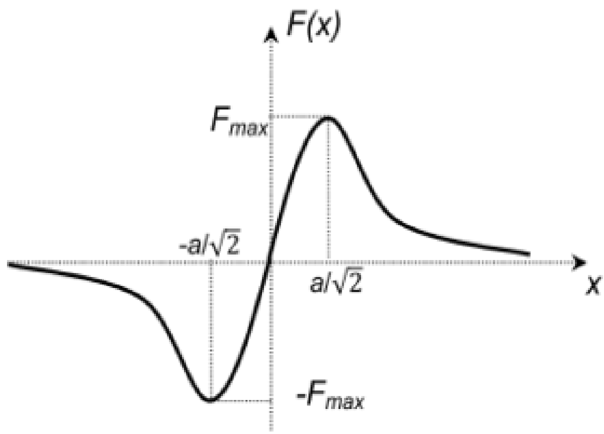


Principe de superposition $\Rightarrow \vec{F} = \vec{F_A} + \vec{F_B}$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q.Q}{r^2}(\vec{u}_A + \vec{u}_B) = \frac{1}{4\pi\varepsilon_0}\frac{q.Q}{r^2}2\cos\alpha\,\vec{i} ,$$

Exprimons r et $\cos\alpha$ en fonction de a et de x : $r = (a^2 + x^2)^{1/2}$; $\cos\alpha = \frac{x}{r}$

$$\Rightarrow \vec{F}(x) = \frac{1}{2\pi\varepsilon_0}\frac{q.Q}{r^3}x\vec{i} = \frac{1}{2\pi\varepsilon_0}\frac{q.Q}{\left(a^2 + x^2\right)^{\frac{3}{2}}}x\vec{i}$$



Module de $\vec{F}(x)$: $F(x) = \frac{1}{2\pi\varepsilon_0}\frac{q.Q.x}{\left(a^2 + x^2\right)^{\frac{3}{2}}}$

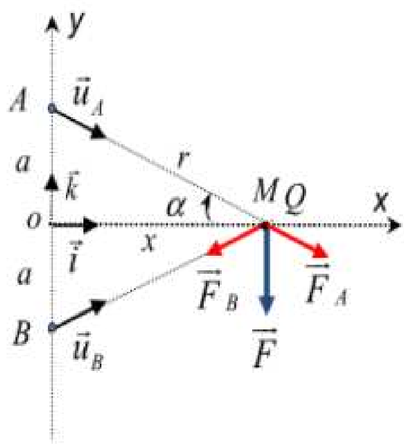
Max de $F(x)$? $\Rightarrow \frac{dF(x)}{dx} = 0 = \frac{q.Q}{2\pi\varepsilon_0}\frac{a^2 - 2x^2}{\left(a^2 + x^2\right)^{\frac{5}{2}}}$

$$\Rightarrow x = \pm\frac{a}{\sqrt{2}} \Rightarrow F_{max} = \pm\frac{q.Q}{\pi\varepsilon_0}\frac{1}{3^{\frac{3}{2}}a^2}$$

A.N : $x_{max} = 21.2\,cm$ et $F_{max} = 0.62\,N$

2) Cas où $q_A = +q$ et $q_B = -q$.

$\vec{F_A} = \frac{1}{4\pi\varepsilon_0}\frac{q.Q}{r_A^2}\vec{u}_A$; $\vec{F_B} = -\frac{1}{4\pi\varepsilon_0}\frac{q.Q}{r_B^2}\vec{u}_B$



Principe de superposition $\Rightarrow \vec{F} = \vec{F_A} + \vec{F_B}$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q.Q}{r^2}(\vec{u}_A - \vec{u}_B) = -\frac{1}{4\pi\varepsilon_0}\frac{q.Q}{r^2}2\sin\alpha\vec{k}$$

$$\sin\alpha = \frac{a}{r} \Rightarrow \vec{F}(x) = -\frac{1}{2\pi\varepsilon_0}\frac{q.Q}{r^3}a\vec{k} = -\frac{1}{2\pi\varepsilon_0}\frac{q.Q}{\left(a^2 + x^2\right)^{3/2}}a\vec{k}$$